**UNIT-I**

**Laplace Transformation**

**DEFINITION:**Let f(t) be a function of t defined for all positive values of t . Then the Laplace Transformationof f(t) , denoted by  and defined as



Provided that the integral exists.Here s is a parameter which may be real or complex.

NOTE: (1) The Laplace Transformation of f(t) exists for s ˃ a, if (i) f(t) is continuous and (ii)  is finite

1. The above condition is only necessary but not sufficient. Ex:  is exists but 

**Transforms of elementary functions:**

1.  (2)and  (3)  and 
2.  and  (5)

**Linear property of Laplace transformation:** If a,b are any constants and f, g are any functions, then



**PROBLEMS:**

1. Find Laplace Transformation of(i)  (ii)  (iii)  (iv)  (v) 

(vi)  (vii)  ( hint:  , )

(viii)  (ix) 

1. Find Laplace Transformation of (i) . (ii)  (iii)  (iv) 

(v) 

1. Find  (A) 
2. Show that 
3. Find Laplace Transformation of  ( A ) 
4. Find Laplace Transformation of  (A) 
5. Find Laplace Transformation of  (A) 
6. Find Laplace Transformation of the following functions:
7. f(t) =  (ii) f(t) =  (iii) f(x) = 

(iv) f(t) =  (v) f(t)=  (vi) f(t) = 

(vii) f(t) = 

1. Find Laplace Transformation of (i) f(t) =  ,t ≥ 0 (ii) f(t) =  where  stands for the greatest integer function. (A) 

**First Shifting property:**Ifthen 

(OR) If then 

**PROBLEMS:**

1. Find Laplace Transformation of (i)  (ii)  (iii)  (iv) 

(v)  (vi)  (vii)  (viii)  (ix) 

1. Find Laplace Transformation of (i)  (ii)  (iii)  (iv) 

**Change of scale property:**If then 

(OR) If  then 

**PROBLEMS:**

1. Find  using change of scale property.
2. If  then find 
3. If , then find 
4. If  then find 

**Laplace transformation of Multiple by tn :**

If  then 

**PROBLEMS:**

Find the Laplace Transformation offollowing functions:

1.  (2)  (3)  (4)  (5)  (6)  (7) 
2.  (8)  (9)  (10)  (11)  (12) 
3.  (15)  (16)  (17)  (18) 

**Laplace transformation of division by t:**

If  then 

**PROBLEMS:**Find the Laplace Transformation of following functions:

1. (2)  (3)  (4)  (5)  (6) 
2.  (8)  (9)  (A: ) (10) 
3.  (12)  (13) (14) 

**Laplace transformation of Derivatives:**Ifand the first n derivatives of f(t) are continuous then 

**PROBLEMS:**

1. Given , then show that 
2. Given , prove that 
3. Find  of the following functions
4. (A: ) (ii)  0 ≤ t ≤ 3 (A:  )

= 6 t ˃ 3

1.  (iv) 
2. Using Laplace transform of derivative find the Laplace transform of the following functions:
3.  (ii) 

**Laplace transformation of Integrals:**

If  then 

**PROBLEMS:**Find the Laplace Transformation of following functions:

1.  (2)  (3)  (4)  (5) 

(6)  (A:  ) (7)  (8)  (9) 

(10)  (11)  (12)  (13)  (14) 

**Evaluation of integrals by Laplace transforms:**

1. A) (7)  A) 
2.  A)  (8)  A) 
3.  A)  (9)  A) 
4.  A)  (10)  A) 
5.  A)  (11) (A) 
6. If  then find  (A) 

**Laplace transforms of periodic functions:**

If f(t) is a periodic function with period T then, 

**PROBLEMS:**

Find the Laplace Transformation of following functions:

1. f(t) = sin wt if 0 <t <

= 0 if < t  (A:  )

1. f(t) = coswt if 0 <t <

= 0 if < t  (A:  )

1. f(t) = t if 0 <t <

=  if < t < 2 (A: 

1. f(t) = t if 0 <t < a

= 2a - t if a < t < 2a (A: )

**Initial Value Theorem:**

If , then 

**Final Value Theorem:**

If , then

**PROBLEMS:**Verify initial and final value theorems for the following functions:

1.  2) 

Verify initial theorems for the following functions:

1.  2) t + sin 3t

Verify final value theorems for the following functions:

1.  (2) 

**Unit step function:**The Unit step function denoted as u(t) and defined as

u(t) = 0 if t < 0

= 1 if t > 0

Note: 

**Heaviside’s Unit step function:**

The Heaviside’s Unit step function denoted as u(t-a) or H(t-a) and defined as

u(t-a) or H(t-a) = 0 if t < a

= 1 if t > a

Note: 

**Second Shifting Theorem:** If then 

(or) If then 

**PROBLEMS:**

1. Find the Laplace Transformation of (A:  )
2. Find the Laplace Transformation of and hence evaluate  (A:  )
3. Find the Laplace Transformation of (A: )
4. Express the following functions in terms of unit step functions and find the Laplace transform:
5. f(t) = t2 0 < t < 1

=4t t > 1 (A:  )

1. f(t) = sin 2t 2< t < 4

=4t otherwise (A: )

1. f(t) = cos t 0 < t < (A:  )

= sin t t>

1. f(t) = cos t 0 < t <(same prob in sin)

=cos 2t < t < 2

= cos 3t t > 2

(v) f(t) = t – 1 1 < t < 2

= 3 – t 2 < t 3

(vi) f(t) = 0 0 < t < 1 (A:  )

= t – 1 1 < t < 2

=1 t > 2

**UNIT-II**

**Inverse Laplace Transformation**

**Definition:** If then f(t) is called inverse Laplace transform of  and symbolically written as . Where  is called inverse Laplace transform operator.

**Standerd Results:** Find inverse Laplace Transform of the following functions:

1.  2)  3)  4) 

**First Shifting Theorem:**If then 

1. i)  ii)  iii)  iv)  v) 
2. i)  ii)  iii)  iv) 

**Evaluate Inverse Laplace Transform by Partial Fractions:**

1. i)  ii)  iii)  iv)
2. i)  ii)  iii)
3. i)  (ii)  (iii) (iv) 
4. i)  ii) iii)  iv) 

v) vi) vii) 

5) i)  ii)  iii) (A:  ) iv) 

**Inverse Laplace Transform of Derivatives:** If  then 

**PROBLEMS:** Find ILT of following functions:

(1) (i)  (ii)  (iii)  (iv)  (v) 

(vi)

(2) (i)  (ii)  (iii)  (iv)  (v) 

(3) (i)  (ii)  (iii) 

**Inverse Laplace Transform of multiple by s:**If and f ( 0 ) = 0 then 

**PROBLEMS:** Find ILT of following functions:

(1)  (2)  (3)  (4) (5) 

**Inverse Laplace Transform of division by s:** If  then 

**PROBLEMS:** Find ILT of following functions:

(1)  (2)  (3)  (4)  (5)  (6) 

**Convolution theorem:**If and then 

**PROBLEMS:** Apply Convolution theorem to evaluate

(1)  (2)  (3)  (4)  (5) 

(6)  (7)  (8)  (9) 

**Application to differential equations:**

(1) given and 

(2) with x = 2 ,  at t = 0.

(3) given that 

(4) , x = Dx = 0 at t = 0.

(5) , if x ( 0 ) = 1 , 

(6) , y ( 0 ) = 2 , 

(7) , 

(8) , y (0) = 1, 

(9) when y (0) = 1, 

(10) when y (0) = -3 , 

(11)  ,  when t=0

(12)  , y (0) = 0 ,

(13) , x = Dx = 0 at t = 0

(14) when y (0)=0 , 

(15) when y(0) = 3 , 

**UNIT-III**

**FOURIER SERIES**

**EULER’S FORMULAE:**

The Fourier series for the function f (x) in the interval α˂ x ˂α + 2 is given by



where ;  ; 

These values of  are known as Euler’s formulae.

**Problems:**

(1) Obtain the Fourier series for  in the interval 0 ˂ x ˂ 2

(a) 

(2) Find the Fourier series to represent  from x = - to x = . Deduce that 

(a) 

(3)Expand  as a Fourier series in the interval 0 ˂ x ˂ 2.

(a) ( sameprob as xcos x)

(4) Prove that  , -˂ x ˂. Hence show that

(i) (ii)  (iii)  (iv) 

(5) for -˂ x ˂ and  for . Expand f (x) in Fourier series. Hence show that

and

(6) Expand  , 0 ˂ x ˂ 2 in a Fourier series. Hence evaluate 

(a) ; put x = 0.

(7) If  in the rang 0 to 2 , show that  (put x = 0)

(8) Express  as Fourier series in the interval -˂ x ˂.

(a) 

(8) Find the Fourier series to represent  in the interval ( 0 , 2 )

(a) 

(9) Find the Fourier series to represent ,0˂ x ˂ 2. Sketch the graph of f (x) from - to .

(a) 

(10) Find the Fourier series to represent the function from x= - to x=  . Deduce from this that



**FOURIER SERIES OF THE FUNCTION HAVING POINTS OF DISCONTINUITY:**

(1) Find the Fourier series expansion for f ( x ) , if f (x) = , ˂x˂0 and f (x) =x , 0 ˂ x ˂. Hence deduce that 

(2) If f (x) = 0 ,˂x˂0 and f (x) = sin x , 0 ˂ x ˂ , prove that  . Hence show that 

(3) Find the Fourier series for the function f (t) = -1 for -˂t ˂ , f (t) =0 ˂t ˂ , f (t) = 1 , ˂t˂ 

**UNIT-IV**

**Z-TRANSFORMS**

**Definition:**If the function f(n) is defined for discrete values n= 0,1,2,…. and f(n) = 0 for n ˂ 0, then its Z- transform is defined to be



whenever the infinite series converges. The inverse Z-transform is written as 

**Some standard Z-transforms:**

1.  (2)  (3) 
2.  (5)  (6) 
3. 

(8) 

(9) Linear property

(10) Change of scale Property or damping rule :

If , then  or 

(11)  , 

(12) 

(13) 

**PROBLEMS:**

(1) Find Z- transform of the following:

(i)  (ii)  ( A  ) (iii) 

(iv)  (A:  (v) ( A:  )

(2) Find Z- transform (i) (ii)  (A :,

(3) Find Z- transform of (i)  (ii)  (A:,

(4) Find Z- transform of (i)  (ii) 

(A:  ,  )

**Shifting properties:**

**(1) Right shifing property:** If  , then 

**(2) Left shifing property:** If  , then 

**Note:** (1)  (2) 

**(3)** 

**PROBLEMS:**

(1) Evaluate ( A: 

(2) Show that 

(3) Find (i)  , (ii)  (iii) 

(A: , )

**Initial value theorem:** If  then 

**Note:** ,  , and so on.

**Final value theorem:**If  then 

**PROBLEMS:**

1) If , evaluate f(2) and f(3). ( 0,0,2,13 )

2) If  show that f(1)= 2, f(2) = 21, f(3) = 139.

3) If  , find the values of u2 and u3 (2, 11)

4) If , find the z-transform of (A:  )

**Inverse z-transform:**

|  |  |  |
| --- | --- | --- |
| S.No. | F(z) |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |

**Evaluation of inverse Z-Transforms by partial fractions:**

**PROBLEMS:**Find the inverse Z-transforms of the following functions:

1)  (A:  )

2)  (A: )

3)  (A:  )

4)  (A:  )

5)  (A:  )

6)  (A: 1+2n )

**Evaluation of inverse Z-Transforms by residue theorem:**

The inverse Z-transform of F(z) is given by

 = sum of the residues of  at its poles.

**PROBLEMS::** Find the inverse Z-transforms of the following functions:

1)  (A:  ) (2)  (A:  )

3)  4)  5)  6) 

7)  8) 

**Convolution theorem:** If  and  then



**PROBLEMS:** Use Convolution theorem finds the inverse Z-transform of the following functions.

1)  (A:  ) 2) 

**Applications of Z-transforms:** Using Z-transforms solve the following differential equations:

(1) with ,  (A) 

(2) with (A) 

(3) with  ,  and  for n = 0,1,2,3…..

(A) 

(4)  (A) 

(5)  , y(0) = Y(1) = 1

(6) y(0) = Y(1) = 0

(7) y0=0 , y1= 2

(8) , n ≥ 2, f(0) = 3 , f(1) =-2

(9)  , y( 0) = 4, y( 1) =0, y( 2) =8

(10) 

(11)  ,  , 

(12) 

(13) 